Druge laboratorijske vežbe iz TEKa - prvi zadatak

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1 Redukovani tablo jednačina

1.1 SKZ+KR

1:

$$i_L(t) = -\frac{u(t)}{R}$$

2:

$$C\frac{du_C(t)}{dt} = \beta i_L(t)$$

1.2 NKZ+KR

I:

$$u_g(t) + L\frac{di_L(t)}{dt} = u(t)$$

1.3 Dimenzije

Pošto graf ima 5 grana, i svakoj grani pridružujemo po napon i struju, sledi da je maksimalan broj jednačina 10. Te jednačine dolaze sa 3 mesta:

- Konstutivnih relacija kojih ima 5 (za svaku granu).
- Strujnog Kirhofovog zakona, jednačina ima N-1 (N je broj čvorova).
- Naponskog Kirhofovog zakona, jednačina ima E-N+1 (E je broj grana).

Maksimalni tablo: 10 jednačina x 10 nepoznatih.
Redukovani tablo: 3 jednačine x 3 nepoznate.

2 Jednačine stanja i izlazna jednačina

2.1 JS

$$\begin{bmatrix} \frac{du_C(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\beta}{C} \\ 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} u_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L} \end{bmatrix} \begin{bmatrix} u_g(t) \end{bmatrix}$$

$$\underline{\dot{x}} = \underset{\sim}{A}\underline{x} + \underset{\sim}{B}\underline{w}$$

2.2 IJ

$$\begin{bmatrix} u(t) \end{bmatrix} = \begin{bmatrix} 0 & -R \end{bmatrix} \begin{bmatrix} u_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} u_g(t) \end{bmatrix}$$

$$\underline{y} = \underset{\sim}{C}\underline{x} + \underset{\sim}{D}\underline{w}$$

3 Rešavanje JS+IJ

3.1 Leva strana RUI

$$P_2[\underline{s}] = det(\underline{s}I - \underset{\sim}{A}) = \begin{vmatrix} \underline{s} & -\frac{\beta}{C} \\ 0 & \underline{s} + \frac{R}{L} \end{vmatrix} = \underline{s}(\underline{s} + \frac{R}{L}) = \underline{s}^2 + \frac{R}{L}\underline{s}$$

Sledi da leva strana RUI iznosi:

$$\underline{\ddot{y}} + \frac{R}{L}\underline{\dot{y}}$$

3.2 Desna strana RUI

$$\underline{y} = \underset{\sim}{C}\underline{x} + \underset{\sim}{D}\underline{w} = \begin{bmatrix} 0 & -R \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} \underline{w}$$

$$\underline{\dot{y}} = \underset{\sim}{C}\underline{\dot{x}} + \underset{\sim}{D}\underline{\dot{w}} = \underset{\sim}{C}(\underset{\sim}{A}\underline{x} + \underset{\sim}{B}\underline{w}) + \underset{\sim}{D}\underline{\dot{w}} = \underset{\sim}{C}\underset{\sim}{A}\underline{x} + \underset{\sim}{C}\underset{\sim}{B}\underline{w} + \underset{\sim}{D}\underline{\dot{w}}$$

$$= \begin{bmatrix} 0 & -R \end{bmatrix} \begin{bmatrix} 0 & \frac{\beta}{C} \\ 0 & -\frac{R}{L} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & -R \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{1}{L} \end{bmatrix} \underline{w} + \begin{bmatrix} 0 \end{bmatrix} \underline{\dot{w}}$$

$$= \begin{bmatrix} 0 & \frac{R^2}{L} \end{bmatrix} \underline{x} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{w} + \begin{bmatrix} 0 \end{bmatrix} \underline{\dot{w}}$$

$$\ddot{y} = \underset{\sim}{C}\underset{\sim}{A}\underline{\dot{x}} + \underset{\sim}{C}\underset{\sim}{B}\underline{\dot{w}} + \underset{\sim}{D}\underline{\ddot{w}} = \underset{\sim}{C}\underset{\sim}{A}(\underset{\sim}{A}\underline{x} + \underset{\sim}{B}\underline{w}) + \underset{\sim}{C}\underset{\sim}{B}\underline{\dot{w}} + \underset{\sim}{D}\underline{\ddot{w}}$$

$$= \underset{\sim}{C}\underset{\sim}{A}\underset{\sim}{A}\underline{x} + \underset{\sim}{C}\underset{\sim}{A}\underset{\sim}{B}\underline{w} + \underset{\sim}{C}\underset{\sim}{B}\underline{\dot{w}} + \underset{\sim}{D}\underline{\ddot{w}}$$

$$= \begin{bmatrix} 0 & \frac{R^2}{L} \end{bmatrix} \begin{bmatrix} 0 & \frac{\beta}{C} \\ 0 & -\frac{R}{L} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & \frac{R^2}{L} \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{1}{L} \end{bmatrix} \underline{w} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{\dot{w}} + \begin{bmatrix} 0 \end{bmatrix} \underline{\ddot{w}}$$

$$= \begin{bmatrix} 0 & -\frac{R^3}{L^2} \end{bmatrix} \underline{x} + \begin{bmatrix} -\frac{R^2}{L^2} \end{bmatrix} \underline{w} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{\dot{w}} + \begin{bmatrix} 0 \end{bmatrix} \underline{\ddot{w}}$$

Dakle, dobijemo sledeće jednačine, koje treba da ubacimo u levu stranu RUI:

$$\dot{y} = \begin{bmatrix} 0 & \frac{R^2}{L} \end{bmatrix} \underline{x} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{w} + \begin{bmatrix} 0 \end{bmatrix} \underline{\dot{w}}$$

$$\ddot{y} = \begin{bmatrix} 0 & -\frac{R^3}{L^2} \end{bmatrix} \underline{x} + \begin{bmatrix} -\frac{R^2}{L^2} \end{bmatrix} \underline{w} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{\dot{w}} + \begin{bmatrix} 0 \end{bmatrix} \underline{\ddot{w}}$$

$$RUI : \underline{\ddot{y}} + \frac{R}{L}\underline{\dot{y}} =$$

$$= \begin{bmatrix} 0 & -\frac{R^3}{L^2} \end{bmatrix} \underline{x} + \begin{bmatrix} -\frac{R^2}{L^2} \end{bmatrix} \underline{w} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{\dot{w}} + \begin{bmatrix} 0 \end{bmatrix} \underline{\ddot{w}}$$

$$+ \begin{bmatrix} 0 & \frac{R^3}{L^2} \end{bmatrix} \underline{x} + \begin{bmatrix} \frac{R^2}{L^2} \end{bmatrix} \underline{w} + \begin{bmatrix} 0 \end{bmatrix} \underline{\dot{w}}$$

$$= \begin{bmatrix} 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} \underline{w} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{\dot{w}} + \begin{bmatrix} 0 \end{bmatrix} \underline{\ddot{w}}$$

3.3 RUI

Konačno, uvrštavanjem leve i desne strane, dobijemo RUI:

$$\underline{\ddot{y}} + \frac{R}{L}\underline{\dot{y}} = \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{\dot{w}}$$

4 Početni uslovi

$$\underline{y} = \begin{bmatrix} 0 & -R \end{bmatrix} \underline{x}$$

$$\dot{y} = \begin{bmatrix} 0 & \frac{R^2}{L} \end{bmatrix} \underline{x} + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{w}$$

Početni uslovi se dobijaju kada uvrstimo t=0_-:

$$\underline{y}(0_-) = \begin{bmatrix} 0 & -R \end{bmatrix} \underline{x}(0_-)$$

$$\dot{y}(0_-) = \begin{bmatrix} 0 & \frac{R^2}{L} \end{bmatrix} \underline{x}(0_-) + \begin{bmatrix} \frac{R}{L} \end{bmatrix} \underline{w}(0_-)$$

U zadatku su nam date početne vrednosti za $\underline{\text{x}}$, dok nam je data funkcija $\underline{\text{w}}$ pa možemo izračunati početnu vrednost.

$$\underline{x}(0_-) = \begin{bmatrix} u_C(0_-) \\ i_L(0_-) \end{bmatrix} = \begin{bmatrix} U \\ I \end{bmatrix}$$

$$\underline{w}(0_-) = \begin{bmatrix} u_g(0_-) \end{bmatrix} = \begin{bmatrix} Ue^{-\frac{0_-}{T}} \end{bmatrix} = \begin{bmatrix} U \end{bmatrix}$$

Sledi:

$$\underline{y}(0_-) = \begin{bmatrix} -RI \end{bmatrix}$$

$$\underline{\dot{y}}(0_-) = \begin{bmatrix} \frac{R(RI+U)}{L} \end{bmatrix}$$

5 Nalaženje rešenja RUI

5.1 Homogeni deo

Rešenja karakterističnog polinoma $P_2[\underline{s}] = \underline{s}^2 + \frac{R}{L}\underline{s}$ su $\underline{s}_1 = 0$ i $\underline{s}_2 = -\frac{R}{L}$.

Sledi da homogena jednačina ima sledeći oblik:

$$y_h(t, K_1, K_2) = K_1 + K_2 e^{-\frac{R}{L}t}$$

5.2 Partikularni deo

Ulazni signal $u_g(t)$ je oblika $Ue^{-\frac{t}{T}}$, odnosno $s_f = -\frac{1}{T}$.

Ako je $s_f = s_2$, imamo rezonanciju, pa će partikularno rešenje biti oblika:

$$y_p(t, A) = Ate^{-t/T}$$

U suprotnom, partikularno rešenje će biti oblika:

$$y_p(t, A) = Ae^{-\frac{t}{T}}$$

5.2.1 Rezonancija, $T = \frac{L}{R}$

$$y_p(t, A) = Ate^{-\frac{t}{T}}$$

$$\dot{y}_p(t, A) = Ae^{-\frac{t}{T}} - \frac{A}{T}te^{-\frac{t}{T}}$$

$$\ddot{y}_p(t, A) = -\frac{2A}{T}e^{-\frac{t}{T}} + \frac{A}{T^2}te^{-\frac{t}{T}}$$

$$u_g(t) = Ue^{-\frac{t}{T}}$$

$$\dot{u}_g(t) = -\frac{U}{T}e^{-\frac{t}{T}}$$

Uvrštavanjem u RUI dobijemo:

$$\ddot{y}_p(t, A) + \frac{R}{L}\dot{y}_p(t, A) = \ddot{y}_p(t, A) + \frac{1}{T}\dot{y}_p(t, A) = \frac{R}{L}\dot{u}_g(t) = \frac{1}{T}\dot{u}_g(t)$$

$$-\frac{2A}{T}e^{-\frac{t}{T}} + \frac{A}{T^2}te^{-\frac{t}{T}} + \frac{A}{T}e^{-\frac{t}{T}} - \frac{A}{T^2}te^{-\frac{t}{T}} = -\frac{U}{T^2}e^{-\frac{t}{T}}$$

$$A = \frac{U}{T}$$

Konačno, partikularni deo je sledeći:

$$y_p(t) = \frac{U}{T}te^{-\frac{t}{T}}$$

5.2.2 Nema rezonancije, $T \neq \frac{L}{R}$

$$y_p(t, A) = Ae^{-\frac{t}{T}}$$

$$\dot{y_p}(t, A) = -\frac{A}{T}e^{-\frac{t}{T}}$$

$$\ddot{y_p}(t, A) = \frac{A}{T^2}e^{-\frac{t}{T}}$$

$$u_g(t) = Ue^{-\frac{t}{T}}$$

$$\dot{u}_g(t) = -\frac{U}{T}e^{-\frac{t}{T}}$$

Uvrštavanjem u RUI dobijemo:

$$\ddot{y_p}(t, A) + \frac{R}{L}\dot{y_p}(t, A) = \frac{R}{L}\dot{u}_g(t)$$

$$\frac{A}{T^2}e^{-\frac{t}{T}} - \frac{AR}{LT}e^{-\frac{t}{T}} = -\frac{UR}{LT}e^{-\frac{t}{T}}$$

$$\frac{A}{T} - \frac{AR}{L} = -\frac{UR}{L}$$

$$A\frac{L - RT}{LT} = -\frac{UR}{L}$$

$$A = -\frac{URT}{L - RT}$$

Konačno, partikularni deo je sledeći:

$$y_p(t) = \frac{URT}{L - RT}e^{-\frac{t}{T}}$$

5.3 Nalaženje konstanti homogenog dela

5.3.1 Rezonancija, $T = \frac{L}{R}$

$$\begin{aligned}
y(t, K_1, K_2) &= y_p(t) + y_h(t, K_1, K_2) \\
&= \frac{U}{T}te^{-\frac{t}{T}} + K_1 + K_2e^{-\frac{R}{L}t} \\
&= \frac{U}{T}te^{-\frac{t}{T}} + K_1 + K_2e^{-\frac{t}{T}} \\
\dot{y}(t, K_1, K_2) &= -\frac{U}{T^2}te^{-\frac{t}{T}} + \frac{U}{T}e^{-\frac{t}{T}} - \frac{K_2}{T}e^{-\frac{1}{T}t}
\end{aligned}$$

Konstante nalazimo iz početnih uslova:

$$y(0_-) = -RI$$

$$\dot{y}(0_-) = \frac{R(RI + U)}{L} = \frac{RI + U}{T}$$

Rešavanjem dobijamo:

$$+K_1 + K_2 = -RI$$

$$\frac{U}{T} - \frac{K_2}{T} = \frac{RI + U}{T}$$

$$K_2 = -RI$$

$$K_1 = 0$$

Konačno rešenje RUI:

$$y(t) = u(t) = \frac{U}{T} t e^{-\frac{t}{T}} - RIe^{-\frac{t}{T}}$$

5.3.2 Nema rezonancije, $T \neq \frac{L}{R}$

$$y(t, K_1, K_2) = y_p(t) + y_h(t, K_1, K_2)$$

$$= \frac{URT}{L - RT} e^{-\frac{t}{T}} + K_1 + K_2 e^{-\frac{R}{L}t}$$

$$\dot{y}(t, K_1, K_2) = -\frac{UR}{L - RT} e^{-\frac{t}{T}} - \frac{K_2 R}{L} e^{-\frac{R}{L}t}$$

Konstante nalazimo iz početnih uslova:

$$y(0_-) = -RI$$

$$\dot{y}(0_-) = \frac{R(RI + U)}{L}$$

Rešavanjem dobijamo:

$$\frac{URT}{L - RT} + K_1 + K_2 = -RI$$

$$-\frac{UR}{L - RT} - \frac{K_2 R}{L} = \frac{R(RI + U)}{L}$$

$$K_2 = -\frac{UL}{L - RT} - RI - U$$

$$K_1 = U + \frac{U(RT - L)}{L - RT} = U - U = 0$$

Konačno rešenje RUI:

$$y(t) = u(t) = \frac{URT}{L - RT} e^{-\frac{t}{T}} + (-\frac{UL}{L - RT} - RI - U)e^{-\frac{R}{L}t}$$